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Radiation transfer in ocean water: Average penetration depth of a photon flux in a fixed time interval

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[1] It has been shown that for a set of photons emitted, initially as a parallel flux, within the water, the average distance traversed along the original direction by the surviving photons in time interval Δt is $\overline{\Delta r} = d\ \exp(-\ bd)\sum_{\infty}^{n=0}\frac{(bd)^n}{(n+1)!}\left[\sum_{n}^{i=0}\bar{\mu}_s^i\right]$ or, $\overline{\Delta r} = d\ F(b,\ \bar{\mu}_s,\ d)$ where d is $c_w\Delta t$ (c_w being the speed of light in water), b is the scattering coefficient, $\bar{\mu}_s$ is the average cosine (or asymmetry factor) of the scattering phase function and $F(b,\ \bar{\mu}_s,\ d)$ may be referred to as the penetration function. The average depth traversed in time interval Δt by those photons which are emitted from a thin layer of the ocean at depth z, but not absorbed, is $\overline{\Delta z} = c_w\Delta t\ F(b,\ \bar{\mu}_s,\ c_w\Delta t)\ \bar{\mu}(z)$ where $\bar{\mu}\ (z)$ is the average cosine of the light field at that depth. Computer calculations based on these equations can readily be implemented, and the validity of the calculations has been confirmed by Monte Carlo modeling.

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1. Introduction

[2] The downward flow of radiant energy through the water column is controlled by both absorption and scattering of photons in the aquatic medium. The depth profile of radiant energy flow, characterized for example in terms of a vertical attenuation coefficient (K) for irradiance, can be expressed as empirical

$$K_d(av) \approx \frac{1}{\bar{\mu}_o}\left(a^2 + Gab\right)^{\frac{1}{2}}$$

or quasi-theoretical

$$^{w}K_E(av) \approx \frac{a}{\bar{\mu}_o}\left[1 + \frac{b}{a}(1 - \bar{\mu}_s)\right]$$

functions of the absorption (a) and scattering (b) coefficients [*Kirk*, 1984, 1991, 2003] of the water. K_d(av) is the average value of K for downward irradiance (E_d) obtained by linear regression (of ln E_d) from the surface down to, typically, that depth at which E_d is 1% of the surface value. $\bar{\mu}_o$ is the average cosine of the incident light flux just below the water surface. G is the coefficient determining the relative contribution of scattering to attenuation of irradiance. $^{w}K_E$(av) is the irradiance-weighted vertical attenuation coefficient for net downward irradiance. $\bar{\mu}_s$ is the average cosine (or asymmetry factor) of the scattering phase function.

[3] The rate of attenuation with depth is determined interactively by absorption and scattering together. In addition to redirecting some downwelling photons upwards, scattering – by making the light field more diffuse – increases the average path length the photons must traverse in passing through a given depth interval, and so increases the probability of their being absorbed within that depth interval.

[4] The rate of attenuation per unit time or, which is the same thing, per unit photon path length (in any direction) differs from attenuation per unit depth in that the extinction probability is the same for all photons regardless of direction, and is determined only by the absorption coefficient. It is scattering, however, together with angular distribution of the photons at the start of their journey, which determines the vertical distribution of the surviving photons within the water column at the end of the prescribed time interval.

[5] The problem to be addressed here is to ascertain whether, by mathematical analysis of the effects of multiple scattering on the angular distribution of a flux of photons, it is possible to arrive at an expression for the average depth interval traversed by the photons within a specified time interval. In earlier work it was shown that for any set of photons that undergo exactly n scatterings per photon, the average cosine is $\bar{\mu}_o\bar{\mu}_s^n$ where $\bar{\mu}_o$ is the average cosine of the photon flux before scattering and $\bar{\mu}_s$ is the average cosine of the scattering phase function [*Kirk*, 1999]. By taking into account the frequency distribution of scattering events it was further possible to show that for a set of photons which has traversed a total path length d through the water, the average cosine is $\bar{\mu}_o$ exp $[-bd(1 - \bar{\mu}_s)]$.

[6] It is here shown that by application of a similar theoretical approach to the behavior of a set of photons commencing their journey at depth z and time t, with initial

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average cosine $\bar{\mu}$ (z), it is possible to arrive at an expression for the average depth interval, $\overline{\Delta z}$, traversed by the surviving photons in time interval Δt, over total path length $c_w \Delta t$ (where c_w is the speed of light in water). The validity of the theoretical conclusions arrived at have been tested by Monte Carlo computer modeling.

2. Theory

[7] Let us assume that at some point within the ocean a flux of photons is emitted as a thin parallel beam of light at zenith angle θ_i at time t. We shall consider what happens to this photon flux in time interval Δt, during which, if they are not absorbed, they traverse a total path length of $c_w \Delta t$, where c_w is the speed of light in water. For simplicity we shall indicate $c_w \Delta t$ by d. We shall, to begin with, confine our attention to those photons which avoid being absorbed in time interval Δt, and we shall examine the effect of scattering on the distance the photons traverse along the original direction of the flux, this distance being indicated by Δr.

[8] An unscattered photon travels the full distance, d, along its original direction (θ,ϕ), where θ is the zenith angle and ϕ is the azimuth angle.

$$\Delta r_0 = d$$

Δr_0 being the distance traveled along the original direction of the flux by a photon with 0 scatterings. For a trajectory with one scattering event at angle γ, the distance traversed along direction (θ, φ) is

$$\Delta r_1 = d_0 + d_1 \cos \gamma \tag{1}$$

where d_0 is the length of the initial, unscattered, part of the trajectory and d_1 is the length of the scattered part, ($d_0 + d_1$) being equal to d. All points along d are equally likely for the scattering event to occur, so the average value of d_0 must be $d/2$, and consequently the average value of d_1 is ($d - d/2$), which again is $d/2$. We already know (see above) that for any set of photons that undergo exactly n scatterings per photon, the average cosine is $\bar{\mu}_o \, \bar{\mu}_s^n$, where $\bar{\mu}_o$ is the average cosine of the photon flux before scattering and $\bar{\mu}_s$ is the average cosine of the scattering phase function. In the present case the photons before scattering have a cosine of 1.0 relative to their initial direction and so the average value of cos γ for the singly scattered photons is $\bar{\mu}_s$. We can now write an expression for the average value that Δr will have for trajectories in which there is one scattering event.

$$\begin{gathered}\overline{\Delta r_1} = \bar{d}_0 + \bar{d}_1 \overline{\cos \gamma} \\ \overline{\Delta r_1} = \frac{d}{2} + \frac{d}{2} \bar{\mu}_s\end{gathered} \tag{2}$$

[9] We shall now consider trajectories in which there are two or more scattering events. Given an optically homogeneous medium, the probability of a scattering event occurring is the same at all points along the trajectory, i.e., the events are randomly distributed over the length of the trajectory. Thus, if there are two scattering events the average distance between the origin and the first scattering ($\bar{d}_0$), between the first and the second scattering ($\bar{d}_1$), and between the second scattering and the end of the trajectory ($\bar{d}_2$), must be the same and so all are equal to $d/3$. The section of the trajectory which has been scattered once already has an average cosine relative to the original photon direction of $\bar{\mu}_s$. The final section of the trajectory corresponding to photons which have been scattered twice thus has an average cosine (relative to the original direction) of $\bar{\mu}_s^2$, so that for the average value that Δr will have for trajectories in which there are two scattering events we obtain

$$\overline{\Delta r_2} = \bar{d}_0 + \bar{d}_1 \bar{\mu}_s + \bar{d}_2 \bar{\mu}_s^2$$

and

$$\overline{\Delta r_2} = \frac{d}{3} + \frac{d}{3} \bar{\mu}_s + \frac{d}{3} \bar{\mu}_s^2 \tag{3}$$

For trajectories in which there are three scattering events we obtain in a similar manner

$$\overline{\Delta r_3} = \frac{d}{4} + \frac{d}{4} \bar{\mu}_s + \frac{d}{4} \bar{\mu}_s^2 + \frac{d}{4} \bar{\mu}_s^3 \tag{4}$$

which we can write as

$$\overline{\Delta r_3} = \frac{d}{4} \left[1 + \bar{\mu}_s + \bar{\mu}_s^2 + \bar{\mu}_s^3\right] \tag{5}$$

and by an extension of this argument, for a trajectory with n scattering events we obtain

$$\overline{\Delta r_n} = \frac{d}{n+1} \left[1 + \bar{\mu}_s + \bar{\mu}_s^2 + \ldots + \bar{\mu}_s^n\right] \tag{6}$$

If we rewrite this equation as

$$\overline{\Delta r_n} = \frac{d}{n+1} \left[\bar{\mu}_s^0 + \bar{\mu}_s^1 + \bar{\mu}_s^2 + \ldots + \bar{\mu}_s^n\right] \tag{7}$$

then it is easily seen that it can be expressed in the more compact form

$$\overline{\Delta r_n} = \frac{d}{n+1} \sum_{n}^{i=0} \bar{\mu}_s^i \tag{8}$$

[10] Equation (8) tells us the average distance traversed by the photons in a radiant flux, along the original direction of the flux, where all the surviving photons undergo exactly n scattering events in time interval Δt. The number of scattering events which actually occur along any given photon trajectory will in fact vary in a statistical manner determined by the value of the scattering coefficient, b, and the total path length, d. The average distance, Δr, traversed by all the photons along the original direction is determined by the frequency distribution of scattering events per photon

$$\overline{\Delta r} = P_0 \overline{\Delta r_0} + P_1 \overline{\Delta r_1} + P_2 \overline{\Delta r_2} + \ldots + P_j \overline{\Delta r_j} + \tag{9}$$

where P_0, P_1, $P_2 \ldots P_j \ldots$ etc. are the probabilities of 0, 1, 2 ... j ... scatterings along path length d. It was shown

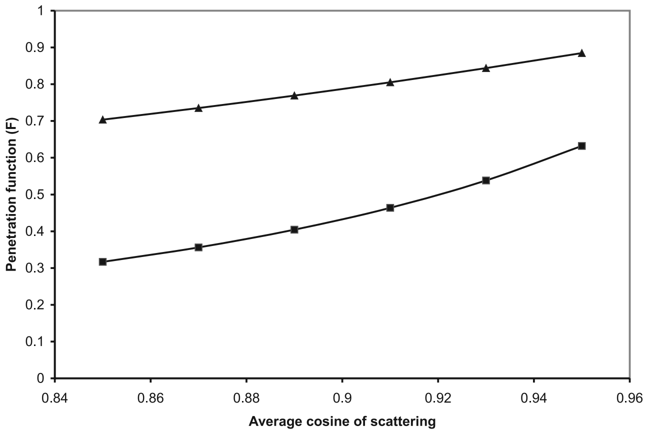

Figure 1. Dependence of the penetration function on the average cosine of the scattering phase function. The values of $F(b, \bar{\mu}_s, d)$ have been calculated using equation (14) for water with scattering coefficient values of 0.05 m^{-1} (solid triangles) and 0.2 m^{-1} (solid squares), and a photon path length (d) of 100 m, over a range of values of $\bar{\mu}_s$.

previously [*Kirk*, 1999] that the proportion of photons that is scattered exactly n times along path length $c_w\Delta t$, i.e., along d, is

$$P_n = \frac{(bd)^n}{n!}\exp(-bd) \quad (10)$$

We can write equation (9) as the sum of an infinite series

$$\overline{\Delta r} = \sum_{\infty}^{n=0} P_n \overline{\Delta r_n} \quad (11)$$

Substituting for P_n and $\overline{\Delta r}_n$ from equations (10) and (8), we obtain

$$\overline{\Delta r} = d\exp(-bd)\sum_{\infty}^{n=0}\frac{(bd)^n}{(n+1)!}\left[\sum_{n}^{i=0}\bar{\mu}_s^i\right] \quad (12)$$

In this way we have arrived at an expression for the average distance traversed by an initially parallel flux of photons in time Δt along their original direction, as a function of two of the IOP of the water, specifically b and $\bar{\mu}_s$. Since, in the derivation of equation (12), no assumptions are made other than that the photon flux is initially parallel, and the water is – over the distance $c_w\Delta t$ – optically homogeneous, this equation is of general applicability. It provides a measure of the foreshortening effect of scattering on a photon flux. Or, to put it in other terms, it is a measure of the extent to which scattering impedes the penetration of the flux through the water. For any total path length, $c_w\Delta t$, the proportionate foreshortening is $\overline{\Delta r}/c_w\Delta t$, or $\overline{\Delta r}/d$. We can thus create a function $F(b, \bar{\mu}_s, d)$, equal to $\overline{\Delta r}/d$, by which any proposed total photon path length, d, can be multiplied to obtain the average photon path length. For water with known scattering characteristics this function enables us to calculate the foreshortening of the flux for any d, i.e.,

$$\overline{\Delta r} = F(b, \bar{\mu}_s, d)d \quad (13)$$

where

$$F(b, \bar{\mu}_s, d) = \exp(-bd)\sum_{\infty}^{n=0}\frac{(bd)^n}{(n+1)!}\left[\sum_{n}^{i=0}\bar{\mu}_s^i\right] \quad (14)$$

[11] This analysis can be extended to a radiant flux in which the photon directions are distributed over all angles. For the set of photons considered above, which have traveled an average distance $\overline{\Delta r}$ along θ_i, the average change in depth is

$$\overline{\Delta z}(\theta_i) = \overline{\Delta r}\cos\theta_i \quad (15)$$

Consider now the light field within the ocean at depth z. The average cosine of the field is $\bar{\mu}(z)$. The photons emitted from a thin layer at time t thus have an average cosine of $\bar{\mu}(z)$. It can readily be shown that for these photons the average change in depth in time Δt is

$$\overline{\Delta z} = \overline{\Delta r}\bar{\mu}(z) \quad (16)$$

or, to express the conclusion more completely, the average depth traversed in time interval Δt by those photons which are emitted from a thin layer of the ocean at depth z but not absorbed is

$$\overline{\Delta z} = c_w\Delta t F(b, \bar{\mu}_s, c_w\Delta t)\bar{\mu}(z) \quad (17)$$

where $F(b, \bar{\mu}_s, c_w\Delta t)$ has the value expressed in equation (14).

3. Calculations

[12] Although equation (12) incorporates an infinite series, this converges quickly and calculation of $\overline{\Delta r}$, $\overline{\Delta z}$ and F can readily be implemented on a personal computer. I have written a Fortran program, *Deltazed*, which carries this out. To ensure that convergence has been achieved, the computer iterates the calculation over a five-fold range of values of n, the number of scattering events per trajectory whose effects are determined. It is typically the case that where the lowest value assigned to n is 10, the last three calculated values of $\overline{\Delta r}$ in the series of five are identical.

[13] It is clear from equation (14) that the value of $F(b, \bar{\mu}_s, c_w\Delta t)$ must decrease, i.e., the extent of the foreshortening must increase, as the value of $\bar{\mu}_s$, the average cosine of the phase function, decreases. This is as might be expected: the wider the angles at which scattering occurs (the lower the value of $\bar{\mu}_s$), the more the penetration of the light flux is impeded. *Petzold* [1972] measured the volume scattering function of a number of ocean waters, and from his data the corresponding values of $\bar{\mu}_s$, the average cosine of scattering, can be calculated [*Kirk*, 1991]: they vary from 0.867 at one station near the Bahama Islands to 0.947 at an offshore Southern California station. To illustrate the extent to which the foreshortening of a photon flux varies with the shape of the phase function at a given level of scattering in typical ocean waters, Figure 1 shows the value of $F(b, \bar{\mu}_s, c_w\Delta_t)$ calculated for water with scattering coefficient values of 0.05 m^{-1} and 0.2 m^{-1}, and making $c_w\Delta t$, the maximum photon path length, equal to 100 meters. It can be seen that,

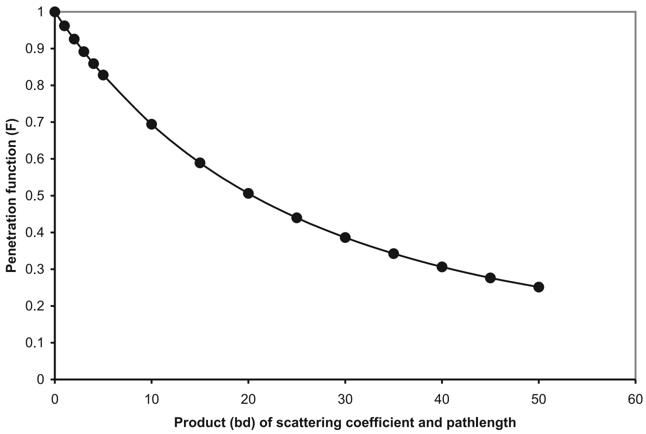

Figure 2. Dependence of the penetration function on the product, *bd*, of the scattering coefficient and the total photon path length. $F(b, \bar{\mu}_s, d)$ values were calculated using equation (14).

to a reasonable approximation, *F* increases linearly with $\bar{\mu}_s$ at the lower scattering value, but in a slightly curvilinear manner at the higher.

[14] It is also clear from equation (14) that although the penetration function is determined by both the scattering coefficient (*b*) and the total photon path length (*d*), it is their product, *bd*, which is the centrally important factor. Figure 2 shows that for water with $\bar{\mu}_s = 0.922$ (San Diego Harbor) *F* decreases by 75% as *bd* increases from 0 to 50. A single data set such as this can be applied to a variety of situations: for example, a *bd* value of 5.0 can correspond to a path length of 100 m and scattering coefficient equal to 0.05 m^{-1}, or to 10 m and 0.5 m^{-1}, or any other combination of *b* and *d* which yields $bd = 5.0$.

[15] Equation (17) with equation (14) tells us the average depth of the photons after a prescribed time interval, but does not reveal how they are distributed with depth. To obtain this information, I have used Monte Carlo modeling to determine the final location of photons emitted at a certain depth within a water body, and permitted to travel over a specified total path length. The Fortran program used, *Pendepth*, was developed from similar programs used in earlier modeling studies of the underwater light field [*Kirk* 1981, 1984, 1991]. The input data for each calculation include the inherent optical properties (absorption coefficient, scattering coefficient, scattering phase function) for the water being modeled, together with the angle at which the photons are deemed to be emitted within the water, and the path length of the photons. Since we are only interested in those photons which complete the allotted path length, $c_w \Delta t$, the absorption coefficient is normally assumed to be zero. To avoid the complicating effects of escape through, or reflection at, the air-water interface, the emission depth is chosen to be such that a negligible proportion of the upward-scattered photons reach the surface.

[16] Figure 3 shows the depth distribution of photons which were emitted vertically downwards at 100 m depth, and which have traversed a path length of 100 m, in water with $b = 0.05$ m^{-1}, and the San Diego Harbor phase function. Photons are distributed from near the surface down to 200 m depth but nearly all of them are, as expected, below the emission depth and the distribution has a very sharp peak just above the maximum possible depth. When the scattering coefficient is increased 10-fold, a very different depth distribution is obtained (Figure 4). Now there is a broad distribution centered around ~130 m: a substantial fraction of the photons end up above the emission depth.

[17] The average final depth of the photons calculated by Monte Carlo modeling is in essentially perfect agreement with that predicted by the theoretical analysis. Table 1 compares values of $\overline{\Delta z}$ obtained by the two calculation procedures for scattering coefficients over the range 0.05 – 3.0 m^{-1}, and for $\bar{\mu}(z)$ values of 1.0 and 0.5. The data also show that for higher values of *b*, e.g., $b \geq 0.5$ m^{-1} corresponding to $bd > 50$ when $d = 100$ m, $\overline{\Delta z}$ becomes inversely proportional to *b*. It can be shown why this must be so. The average number of scattering events per photon trajectory ($\bar{n}$) is equal to *bd* [*Kirk*, 1999]. When *bd* is

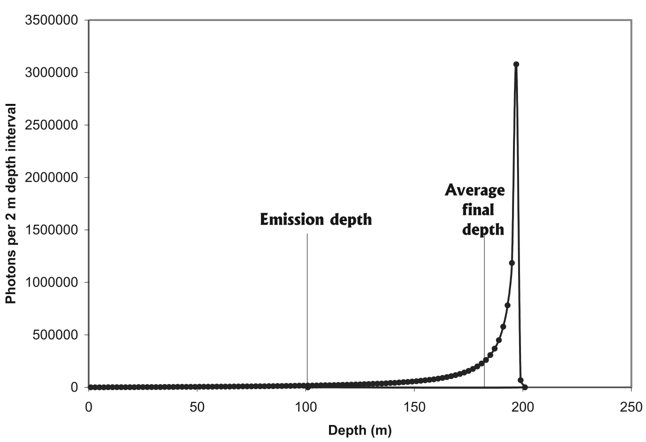

Figure 3. Depth distribution of photons which have traversed total path lengths of 100 m, after being emitted vertically downwards at 100 m depth in water with $b = 0.05$ m^{-1} and the San Diego phase function. Calculated by Monte Carlo modeling.

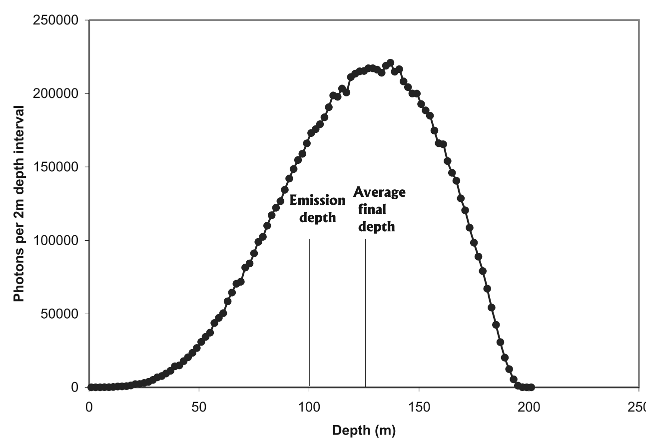

Figure 4. Depth distribution of photons which have traversed total path lengths of 100 m, after being emitted vertically downwards at 100 m depth in water with $b = 0.5$ m^{-1} and the San Diego phase function. Calculated by Monte Carlo modeling.

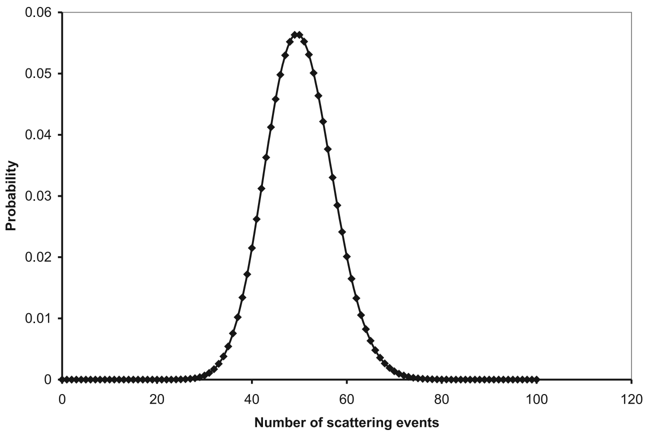

Figure 5. Frequency distribution of number of scattering events per photon path length. The graph shows P_n, the probability of there being precisely n scattering events along the specified path length, as a function of n. The values were calculated with equation (10), assuming $b = 0.5\ \mathrm{m}^{-1}$, and a path length of 100 m.

high, trajectories with high values of n are more probable (Figure 5): for example, when bd is 50, we can calculate using equation (10) that 90% of trajectories have more than 40 scattering events. For large n

$$\sum_{n}^{i=0} \bar{\mu}_s^i \approx \frac{1}{1-\bar{\mu}_s} \tag{18}$$

and modifying equation (12) appropriately we obtain

$$\overline{\Delta r} \approx \frac{d}{1-\bar{\mu}_s} \exp(-bd) \sum_{\infty}^{n=0} \frac{(bd)^n}{(n+1)!} \tag{19}$$

It can readily be shown that

$$\sum_{\infty}^{n=0} \frac{(bd)^n}{(n+1)!} = \frac{\exp(bd)-1}{bd} \tag{20}$$

Substituting into equation (19) and simplifying, we obtain

$$\overline{\Delta r}_{\max} \approx \frac{1}{b(1-\bar{\mu}_s)} \tag{21}$$

and we can also write

$$\overline{\Delta z}_{\max} = \frac{\bar{\mu}(z)}{b(1-\bar{\mu}_s)} \tag{22}$$

Table 1 includes calculated values of $\bar{\mu}(z)/[b(1-\bar{\mu}_s)]$ for each value of scattering coefficient and it can be seen that for high values of bd (~50 upwards), equation (22) satisfactorily predicts $\overline{\Delta z}$. Equations (21) and (22) imply that with increasing bd, $\overline{\Delta r}$ and $\overline{\Delta z}$ do not continue to increase indefinitely but level off at maximum values which are determined entirely by b and $\bar{\mu}_s$. Figure 6 illustrates this for three different values of scattering coefficient. It can be seen that in each case as d (and thus bd) increases, $\overline{\Delta z}$, while first increasing linearly with d, eventually levels off at a maximum value, the values obtained being as predicted by equation (22).

Table 1. Comparison of Average Depth Values Obtained in Monte Carlo Simulations With Those Calculated Using Equations (14) and (17), and Also Equation (22)[a]

Scattering Coefficient, m^{-1}	0.05	0.5	1.0	2.0	3.0
$\bar{\mu}(z) = 1.0$					
$\overline{\Delta z}$, Equations (14) and (17)	82.81	25.12	12.82	6.41	4.27
$\overline{\Delta z}$, Monte Carlo	82.82	25.11	12.79	6.39	4.31
$\overline{\Delta z}$, equation (22)	256.41	25.64	12.82	6.41	4.27
$\bar{\mu}(z) = 0.5$					
$\overline{\Delta z}$, equations (14) and (17)	41.40	12.56	6.41	3.21	2.14
$\overline{\Delta z}$, Monte Carlo	41.40	12.53	6.37	3.18	2.17

[a]Units are in meters. Photon path length is specified to be 100 m, and the water is assumed to have the San Diego phase function ($\bar{\mu}_s = 0.922$). The photons are emitted with $\bar{\mu}(z)$ equal either to 1.0 or 0.5.

4. Discussion

[18] The agreement between the values of $\overline{\Delta z}$ calculated with equations (14) and (17), and those obtained by Monte Carlo modeling is evidence for the validity of the theoretical treatment of the effects of multiple scattering on penetration of a photon flux in the ocean, given in this paper. An immediate inference from these equations is that the only aspect of the scattering phase function that needs to be taken into account is the average cosine of scattering, $\bar{\mu}_s$. The values of the backward and forward scattering probabilities, important though they are in other contexts, are for our present purposes of no significance. As was made clear in the theory section, no account has been taken of the effects of absorption. Given that we have been considering the final situation only of those photons which succeeded in traversing a distance $c_w\Delta t$ from the point of their emission, then

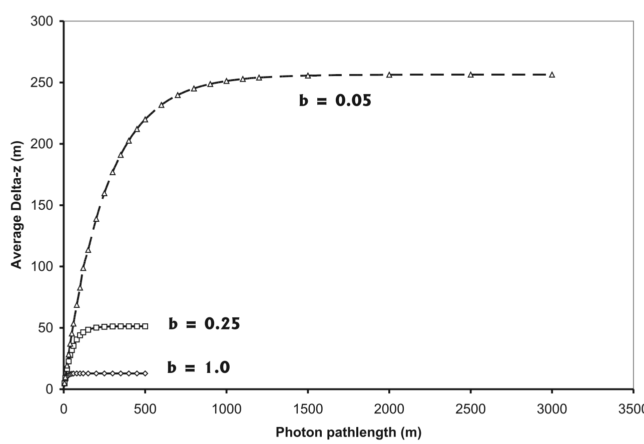

Figure 6. Average penetration depth, $\overline{\Delta z}$, as a function of total path length, for three different values of scattering coefficient: 0.05, 0.25 and 1.0 m^{-1}. The calculation was carried out with equations (14) and (17), assuming the water to have the San Diego phase function ($\bar{\mu}_s = 0.922$), and that the photons are emitted with $\bar{\mu}(z)$ equal to 1.0.

necessarily we disregarded photons which were absorbed along the way. The proportion of emitted photons which survive a path length of $c_w\Delta t$ is simply $exp(-ac_w\Delta t)$. If, instead of examining, as we have, what happens to the photons per unit time elapsed, we consider what happens per unit depth traversed then absorption exerts a major effect on the character of the flux. The more obliquely traveling photons are selectively removed since they have to travel further (greater Δt) to traverse a given depth interval.

[19] The equations derived in the theoretical section of this paper apply exactly to a light flux emitted within the water at depths such that losses through or reflection from the surface are negligible. They should also work well with light emitted through the surface into the sea at locations where scattering is low, and this is in fact the case throughout most of the ocean. Figure 3 shows, for such a water, how nearly all the photons, after time Δt, are below the point of emission. Conversely, we can see from Figure 4 that the equations would not work well for emission down through the surface of a highly turbid water: computer simulation modeling using the Monte Carlo or other techniques would be required for such waters.

[20] An important real-world phenomenon to which the mathematical analysis presented here might be applicable is bioluminescence. This is commonly emitted by any of the enormous variety of marine organisms capable of it, as a brief flash of light consequent upon some disturbing event, whether physical, biological or man-made. The flash consists of a pulse of photons which travel out through the surrounding medium. As it travels it is attenuated by absorption in accordance with $\exp(-ac_w\Delta t)$, and the penetration of the surviving photons through the water in time interval Δt is limited by scattering in the manner prescribed by equations (13) and (14). Another phenomenon to which this study is relevant is the propagation of a pulsed laser beam within the ocean, as used in airborne laser bathymetry [*Sinclair*, 1999] and lidar measurement of concentrations of phytoplankton pigments and colored dissolved organic matter [*Hoge et al.*, 2005].

[21] The finding that for any given water there is a maximum average penetration depth, $\overline{\Delta z}_{max}$, determined (at a given $\bar{\mu}[z]$) only by b and $\bar{\mu}_s$ in accordance with equation (22), seems at first sight counter-intuitive, since we know that photons, for as long as they escape absorption, can always keep on traveling downwards. The explanation is that after a very large number of scatterings the photons have an average orientation close to the horizontal, so those that are still traveling downwards are balanced by those that are traveling upwards.

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