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Light field around a point light source in the ocean

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[1] Bioluminescence in the ocean can, to a reasonable approximation, be regarded as originating in point sources of light. In addition, submerged artificial point sources can be used to measure the absorption coefficients of seawater. To arrive at a better quantitative understanding of these phenomena, a theoretical and computer modeling study has been carried out of the nature of the light field around an underwater point source as determined by the inherent optical properties of the seawater. It has been shown that the radial attenuation coefficient for net outward plane irradiance due solely to absorption and scattering (i.e., setting aside inverse square law attenuation) can satisfactorily be expressed as a function of the IOP by $K_E^{ab}(av) = [a^2 + 1.5(1 - \bar{\mu}_s)ab]^{\frac{1}{2}}$. In addition to providing a predictive tool for the light field around bioluminescent events, this relationship makes possible more accurate determination of absorption coefficients using submerged point sources of light.

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1. Introduction

[2] Bioluminescence is a phenomenon universally present in the world's oceans. The organisms responsible range across the whole biological spectrum, from prokaryotes (bacteria) and eukaryotic unicells (dinoflagellates), through coelenterates (jellies) and crustaceans (copepods, shrimps), to vertebrates (fish). It is a phenomenon of great ecological significance in the marine environment, mainly for avoiding predation at the lower end of the biological scale, or for finding food at the upper end. It is also triggered off by physical disturbance such as the movement of bodies, of biological or non-biological origin, through the underwater medium. To a reasonable approximation the small organisms responsible for most of the bioluminescence in the ocean can be regarded as point sources of light. Man-made point sources of light submerged in the ocean are also of interest because they can be used to measure the absorption coefficients of the water [*Maffione et al.*, 1993; *Maffione and Jaffe*, 1995].

[3] The nature of the light field around a point source of light in the ocean, in particular the manner of its attenuation with distance from the source, is determined by the inherent optical properties (IOP) of the seawater, specifically the absorption coefficient (a), the scattering coefficient (b) and the scattering phase function ($\tilde{\beta}(\theta)$) [*Jerlov*, 1976; *Kirk*, 1994; *Mobley*, 1994]. The aim of the present study was, by using radiative transfer theory together with numerical modeling, to arrive at a mathematical expression describing the nature of the dependence of the attenuation of light around submerged point sources on the inherent optical properties of the water. In this way, it was hoped to devise a simple and useful tool for understanding the light fields created by these ecologically important phenomena.

2. Theory

[4] We shall assume that there exists a point source of light within a homogeneous water body of effectively infinite depth and lateral extent, located at a depth such that the proportion of the emitted light reaching the surface is small enough to be neglected. The source emits light energy equally in all directions. Let the point source be surrounded by a concentric spherical surface of radius r_0, where r_0 is very small: in particular it is small enough so that there is negligible absorption or scattering of light within the sphere. We have thus created a very small spherical source emitting photons uniformly all around its surface, the photons being emitted at right angles to the surface.

[5] As the photons travel out into the surrounding medium some will undergo scattering, so that their direction changes. We define the photon angle as being the angle, θ, between the photon trajectory and the radial axis: the straight line from each photon to the point source. Every trajectory angle has a cosine and so within any small element of volume at radial distance r from the source there is an average value of this cosine - $\bar{\mu}(r)$ - which provides a measure of the angular distribution of the photon population at distance r. On the grounds of radial symmetry this average cosine has the same value at all points at radius r around the source. The photons are all traveling initially at $\theta = 0°$ so that their average cosine on emission, $\bar{\mu}_o$, is equal to 1.0. With increasing radial distance from the source the light field becomes more diffuse so that $\bar{\mu}(r)$ progressively decreases, eventually settling down at the value – determined entirely by the IOP of the water - characteristic of the asymptotic region of the light field.

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[6] The diminution of the irradiance with distance, r, from a point source is due to a combination of two factors: first the attenuation by absorption and scattering processes in the same way as the downward solar radiation stream is attenuated with increasing depth, and secondly, the increase in area over which the outward flux is distributed in accordance with the inverse square law. These two factors are independent and so can be addressed separately. The particular type of irradiance we shall deal with here is the *net* outward irradiance. At any point in the field consider a small planar element of surface perpendicular to the radial line, length r, extending from the point source to the element of surface. On the inner side of this element of surface, i.e., the side facing the source, there is a flux of photons. The irradiance on the element of surface due to this outwardly directed flux, we shall indicate by $E_{out}(r)$. On the outer side of this element of surface, i.e., the side away from the source, there is a smaller flux of photons, originating in backscattering, or multiple forward scattering, at distances further removed from the source. The irradiance on the element of surface due to this inwardly directed flux we shall indicate by $E_{in}(r)$. The *net* outward irradiance on the element of surface we shall, since it is precisely analogous, indicate by the same symbol, $\vec{E}(r)$, as that used for the net downward irradiance of the solar radiation stream

$$\vec{E}(r) = \vec{E}_{out}(r) - \vec{E}_{in}(r). \tag{1}$$

Since we have defined the point source to be isotropic, $\vec{E}(r)$ has the same value at all points in the field with radial distance r from the source. It is with the diminution of net outward irradiance with distance r that we shall be concerned. First we shall consider only that part of the attenuation which is due to absorption and scattering of the photons by the medium, i.e., we shall to begin with ignore the fact that with increasing r, the net outward flux is being spread out over a larger and larger surface.

[7] Over any small increment of distance, Δr, there is a diminution in net outward irradiance, $\Delta\vec{E}(r)$, which is partly due to the inverse square law and partly to absorption and scattering.

$$\Delta\vec{E}(r) = \vec{E}(r+\Delta r) - \vec{E}(r) \tag{2}$$

$$\Delta\vec{E}(r) = {}^{inv}\Delta\vec{E}(r) + {}^{ab}\Delta\vec{E}(r). \tag{3}$$

The *ab* superscript in the second term in equation (3) refers to the contribution of the absorption (a) and scattering (b) coefficients. By correcting for the inverse square law we can convert any value of $\vec{E}(r+\Delta r)$ to the value it would have if the inverse square law did not operate. Let the value that $\vec{E}(r+\Delta r)$ would have if attenuation was due *only* to absorption and scattering be ${}^{ab}\vec{E}(r+\Delta r)$. Then

$${}^{ab}\vec{E}(r+\Delta r) = \vec{E}(r+\Delta r)\frac{(r+\Delta r)^2}{r^2}$$

which, for very small Δr (so that $\Delta r^2 \to 0$) becomes

$${}^{ab}\vec{E}(r+\Delta r) = \vec{E}(r+\Delta r)\left(1+\frac{2r\Delta r}{r^2}\right).$$

The change in irradiance due only to absorption and scattering is ${}^{ab}\Delta\vec{E}(r) = {}^{ab}\vec{E}(r+\Delta r) - \vec{E}(r) = \vec{E}(r+\Delta r)(1+\frac{2\Delta r}{r}) - \vec{E}(r) = \vec{E}(r+\Delta r) - \vec{E}(r) + \vec{E}(r+\Delta r)\frac{2\Delta r}{r}$ which, using equation (2) and rearranging becomes

$$\Delta\vec{E}(r) = {}^{ab}\Delta\vec{E}(r) - \vec{E}(r+\Delta r)\frac{2\Delta r}{r}.$$

Dividing throughout by $\vec{E}(r)\Delta r$

$$\frac{\Delta\vec{E}(r)}{\vec{E}(r)\Delta r} = \frac{1}{\vec{E}(r)}\frac{{}^{ab}\Delta\vec{E}(r)}{\Delta r} - \frac{\vec{E}(r+\Delta r)}{\vec{E}(r)}\frac{2}{r}. \tag{4}$$

Rewriting equation (4) in differential form, and noting that for infinitesimal Δr, $[\vec{E}(r+\Delta r)/\vec{E}(r)] \to 1.0$, we obtain

$$\frac{d\vec{E}(r)}{\vec{E}(r)dr} = \frac{1}{\vec{E}(r)}\frac{{}^{ab}d\vec{E}(r)}{dr} - \frac{2}{r}. \tag{5}$$

[8] Diminution of irradiance with increasing r is, as we have noted, due partly to attenuation caused by absorption and scattering, and partly due to the inverse square law increase in the area irradiated. In accordance with the standard convention for irradiance attenuation coefficients we can define a coefficient for that part of the attenuation of net outward irradiance which is due to absorption and scattering processes.

$$K_E^{ab}(r) = -\frac{1}{\vec{E}(r)}\frac{{}^{ab}d\vec{E}(r)}{dr} \tag{6}$$

Substituting in equation (5) and rearranging, we obtain

$$\frac{d\vec{E}(r)}{\vec{E}(r)} = -K_E^{ab}(r)dr - \frac{2}{r}dr. \tag{7}$$

It is of interest to note at this point that if, invoking the well-known Gershun equation, we replace $K_E^{ab}(r)$ with $a/\overline{\mu}(r)$, then equation (7) becomes

$$\frac{d\vec{E}(r)}{\vec{E}(r)} = -\frac{a}{\overline{\mu}(r)}dr - \frac{2}{r}dr$$

a relationship (their equation (12)) derived by *Maffione et al.* [1993] by a somewhat different route.

[9] Integrating equation (7) from r_o to r

$$\int_{r_o}^{r}\frac{1}{\vec{E}(r)}d\vec{E}(r) = -\int_{r_o}^{r}K_E^{ab}(r)dr - \int_{r_o}^{r}\frac{2}{r}dr$$

we eventually obtain

$$\ln\left[\frac{\vec{E}(r)}{\vec{E}(r_o)}\frac{r^2}{r_o^2}\right] = -\int_{r_o}^{r}K_E^{ab}(r)dr$$

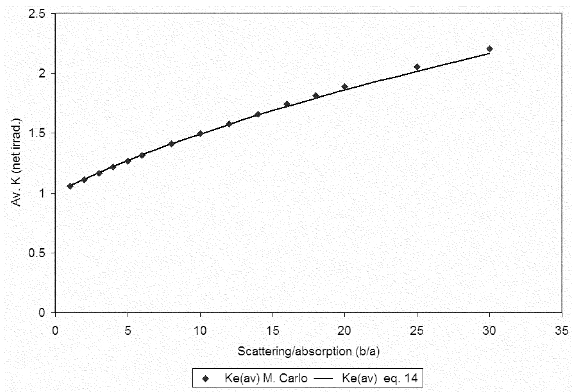

Figure 1. $K_E^{ab}(av)$ as a function of the scattering/absorption (b/a) ratio. The values from the Monte Carlo-modeled light fields (points) are compared with those calculated using equation (14) (continuous line).

from which we arrive at an expression for the net outward irradiance at distance r from the source

$$\vec{E}(r) = \frac{\vec{E}(r_o)}{(r/r_o)^2} \exp\left[-\int_{r_o}^{r} K_E^{ab}(r)dr\right] \quad (8)$$

corresponding to equation (13) of *Maffione et al.* [1993], in which $K_E^{ab}(r)$ is replaced with $a/\overline{\mu}(r)$. The integral in equation (8) divided by the distance over which integration takes place is equal to the average value of $K_E^{ab}(r)$ from r_o to r.

$$\frac{\int_{r_o}^{r} K_E^{ab}(r)dr}{(r - r_o)} = \overline{K_E^{ab}(r)dr} \quad (9)$$

so that we can rewrite equation (8) as

$$\vec{E}(r) = \frac{\vec{E}(r_o)}{\left(r/r_o\right)^2} \exp\left[-\overline{K_E^{ab}(r)}(r - r_o)\right]. \quad (10)$$

[10] Since, as we have already noted, the angular distribution of the light field changes progressively with increasing r in a manner determined by the values of the IOP, we do not in fact know the value of $K_E^{ab}(r)$ for any given r. In the case of the solar radiation field in the ocean, where a similar problem arises due to the changing angular distribution with depth, the normal practice is to determine a depth-averaged value K_d(av), by measurement of irradiance over some particular depth interval, z_1 to z_2, or by linear regression from a set of measurements through the euphotic zone. K_d(av) determined in this way can then be used to calculate irradiance at any depth of interest. Although not exact, this approach has been found very useful and is widely applied. By analogy, then, we might reasonably expect that an equation along the lines of

$$\vec{E}(r) \approx \frac{\vec{E}(r_o)}{(r/r_o)^2} \exp\left[-K_E^{ab}(av)(r - r_o)\right] \quad (11)$$

where $K_E^{ab}(av)$ is some experimentally or theoretically estimated value of the average coefficient for radial attenuation due to absorption and scattering processes might be useful in dealing with the light field around a point source.

3. Monte Carlo Calculations

[11] In the case of the underwater light field created by solar radiation it has been shown, using Monte Carlo modeling [*Kirk*, 1981, 1984, 1991], that K_d(av), the average vertical attenuation coefficient for downward irradiance through the euphotic zone, can quite accurately be expressed as a function of the IOP by the equation

$$K_d(av) \approx \frac{1}{\overline{\mu}_o}\left(a^2 + Gab\right)^{\frac{1}{2}} \quad (12)$$

where $\overline{\mu}_o$ is the average cosine of the refracted solar flux just below the surface, and G is a coefficient whose value depends on $\overline{\mu}_o$ and $\overline{\mu}_s$ (the asymmetry factor, or average cosine, of the scattering phase function). Since, in the far field, K_E(av) is very close in value to K_d(av) we can assume that essentially the same equation applies to the vertical attenuation coefficient for net downward irradiance.

[12] To ascertain whether a similar relationship might apply to the system under study here, a Monte Carlo-based computer model (*Pointsource*) along the lines of those used in the previous work, to calculate the properties of the light field around an underwater point source, has been developed. For each run the values of absorption and scattering coefficients are supplied, and a suitable scattering phase function from the data set of *Petzold* [1972] is chosen. By carrying out preliminary trials with small numbers of photons a radial distance interval which will ensure that the irradiance (inverse square corrected) diminishes to less than 0.01% at 100 intervals is selected, and the properties of the field at each radial interval out to 100 intervals is then calculated, using 10^7 photons. At each radial interval ${}^{ab}\vec{E}(r)$ is determined, and from these values, $K_E^{ab}(av)$ over the radial distance in which ${}^{ab}\vec{E}(r)$ falls to 1% of that at r_o is obtained.

4. Results

[13] For each of the chosen phase functions a series of Monte Carlo runs was carried out (10^7 photons per run) with an absorption coefficient value of 1.0 m^{-1} and scattering absorption ratios (b/a) from 1 to 30. Using a rearranged form of equation (12)

$$G = \frac{a}{b}\left[\left(\frac{K_E^{ab}(av)}{a}\right)^2 - 1\right] \quad (13)$$

in which, since we are dealing with a point source, $\overline{\mu}_o$ has been set equal to 1.0, we obtain the value of the coefficient G corresponding to each $K_E^{ab}(av)$. The average value of G over the b/a range 1 to 30 was then calculated for each phase function.

[14] Figure 1 shows, for a water with the scattering phase function measured by *Petzold* [1972] in the somewhat turbid San Diego Harbour, the variation of $K_E^{ab}(av)$ with $b/$

Table 1. Light Scattering Characteristics of the Different Water Types[a]

Water Type	Average Cosine of Scattering ($\overline{\mu}_s$)	G
1. San Diego Harbour	0.922	0.1230
2. Bahama Is. Station 7	0.915	0.1310
3. Bahama Is. Station 8	0.867	0.1980
4. Bahama Is. Station 9	0.885	0.1768
5. Offshore S. California Station 11	0.947	0.0821
6. Filtered freshwater	0.726	0.3830
7. Seawater	0.929	0.1043
8. Seawater, filtered 40 min	0.931	0.1086
9. Seawater, filtered 100 min	0.918	0.1253
10. Seawater, filtered 18 h	0.660	0.5350

[a] $\overline{\mu}_s$ values calculated [*Kirk*, 1991] from *Petzold* [1972] volume scattering function measurements. The values of the coefficient G, which determines (equation (14)) the contribution of scattering to attenuation, were obtained by Monte Carlo modeling for each water type over a range of b/a values, as described in the text.

a as obtained directly from the Monte Carlo ${}^{ab}\vec{E}(r)$ values (points), and as predicted (continuous line) by

$$K_E^{ab}(av) = \left(a^2 + Gab\right)^{\frac{1}{2}} \tag{14}$$

where G for this particular phase function has been estimated from the data to be 0.123. The agreement is good. Similar findings are obtained with other scattering phase functions from the *Petzold* [1972] data set, although as found previously [*Kirk*, 1991], the value of G varies with the shape of the function. The dependence of K on a and b thus seems to have the same general form for the light field around a point source as it does for the light field established within the ocean under the solar radiant stream.

[15] To characterize the nature of the relationship between G and the shape of the scattering phase function, the value of this coefficient has been determined for a further nine phase functions from the *Petzold* [1972] data set in the same manner as described for the San Diego phase function, above. The calculated G values for all ten phase functions are presented in Table 1, together with the corresponding values of $\overline{\mu}_s$, the average cosine of scattering for each phase function, and $(1 - \overline{\mu}_s)$. Figure 2 shows that G can satisfactorily be represented as a linear function of $(1 - \overline{\mu}_s)$ in accordance with

$$G = 1.502(1 - \overline{\mu}_s).$$

Given the statistical nature of the calculation of G, it cannot confidently be claimed that the coefficient, 1.502, in this equation is accurate to three places of decimals: it therefore seems reasonable and prudent to round it down to 1.50, so that the equation can be written

$$G = 1.5(1 - \overline{\mu}_s). \tag{15}$$

Thus we can now write

$$K_E^{ab}(av) = \left[a^2 + 1.5(1 - \overline{\mu}_s)ab\right]^{\frac{1}{2}}. \tag{16}$$

[16] If, following *Maffione et al.* [1993], we write the net outward irradiance at r_o as

$$\vec{E}(r_o) = \frac{\Phi(r_o)}{4\pi r_o^2} \tag{17}$$

where $\Phi(r_o)$ is the total radiant power at r_o, then substituting for $\vec{E}(r_o)$ and for $K_E^{ab}(av)$ in equation (11) we obtain

$$\vec{E}(r) \approx \frac{\Phi(r_o)}{4\pi r^2} \exp\left\{-\left[a^2 + Gab\right]^{1/2}(r - r_o)\right\} \tag{18}$$

which is the equation for a scattering/absorbing medium corresponding to the *Maffione et al.* [1993] equation (16) for an absorbing/nonscattering medium

$$\vec{E}(r) \approx \frac{\Phi(r_o)}{4\pi r^2} \exp\{-a(r - r_o)\}. \tag{19}$$

Since our light source is very small in relation to the distances over which measurements will be made, i.e., since in general $r_o \ll r$, then we can replace $(r - r_o)$ in equation (18) with r.

$$\vec{E}(r) \approx \frac{\Phi(r_o)}{4\pi r^2} \exp\left\{-\left[a^2 + Gab\right]^{1/2} r\right\} \tag{20}$$

or

$$\vec{E}(r) \approx \frac{\Phi(r_o)}{4\pi r^2} \exp\left\{-\left[a^2 + 1.5(1 - \overline{\mu}_s)ab\right]^{1/2} r\right\}. \tag{21}$$

In this way we have obtained an expression for the irradiance at any point in the water around the source entirely as a function of distance and the inherent optical properties of the medium.

[17] The relationship encapsulated in equation (21) is of general significance, applying as it does to the light field generated in the ocean by any of the innumerable point sources that occur naturally, or to a man-made point source.

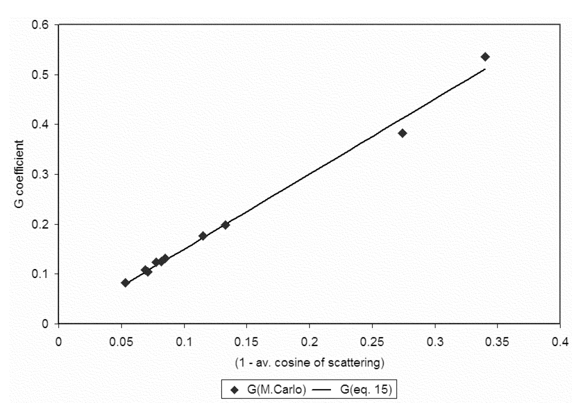

Figure 2. The coefficient G in equation (14) expressed as a function of the average cosine of scattering, (in the form $([1 - \overline{\mu}_s])$. The values obtained from the Monte Carlo-modeled light fields (points) are compared with those calculated using equation (15) (continuous line).

We shall now examine the implications of this relationship for the measurement of absorption coefficients in the ocean using a submerged point source, as carried out for example by *Maffione et al.* [1993]. Following *Maffione et al.* [1993], we multiply throughout by r^2 and take the natural logarithm, to obtain

$$\ln\left[r^2\vec{E}(r)\right] = \ln\left[\frac{\Phi(r_o)}{4\pi}\right] - \left[a^2 + 1.5(1-\bar{\mu}_s)ab\right]^{\frac{1}{2}}r. \qquad (22)$$

To make use of this relationship, we need a set of underwater measurements of net outward irradiance, $\vec{E}(r)$, at a series of distances, r, from an isotropic point source at a depth such that a negligible proportion of the light from the source is lost through the surface. A plot of $\ln[r^2\vec{E}(r)]$ versus r should give a straight line with slope $-[a^2 + 1.5(1 - \bar{\mu}_s)ab]^{\frac{1}{2}}$. *Maffione et al.* [1993] made the simplifying assumption, approximately applicable to clear ocean water, that attenuation could be attributed entirely to absorption, so that the slope of the graph directly yielded the value of a.

[18] To make use of equation (22), the value of the scattering coefficient, b, for the water in question is needed. A reasonable estimate can usually be obtained from the measured beam attenuation coefficient. The average cosine of scattering, or asymmetry factor, $\bar{\mu}_s$, is not so easily obtained but can of course be derived from the volume scattering function, $\beta(\theta)$, if this has been measured. In the absence of $\beta(\theta)$ data, an appropriate value of $\bar{\mu}_s$ can be selected from the set of values calculated (*Kirk* [1991] and Table 1) from the $\beta(\theta)$ measurements of *Petzold* [1972] on a range of water types. We can as an example apply this approach to the data of *Maffione et al.* [1993] Their measurements were carried out in waters off the coast of southern California: they estimated b to have a value of 0.043 m^{-1}. The *Petzold* [1972] $\beta(\theta)$ data for two locations in southern California waters both give rise to $\bar{\mu}_s = 0.947$ m^{-1}. The *Maffione et al.* [1993] graph had a negative slope of 0.0337 m^{-1}, which they took as an estimate of the value of a (at 465 nm). If, in accordance with the theoretical treatment given in this paper, and equation (22), we assume that the slope is in fact equal to $-[a^2 + 1.5(1 - \bar{\mu}_s)ab]^{\frac{1}{2}}$, then we can write

$$0.0337 = \left[a^2 + 1.5(1-\bar{\mu}_s)ab\right]^{\frac{1}{2}}$$

from which, substituting for b and $\bar{\mu}_s$, squaring both sides, and solving the resulting quadratic, we obtain

$$a = 0.032\,\mathrm{m}^{-1}.$$

5. Discussion

[19] On the basis of the theoretical treatment presented in this paper we have arrived at an expression (equation (21) or equation (22)), which describes how the light emitted by natural or man-made point sources in the ocean is attenuated as it moves out from the source, as a function of the inherent optical properties of the water. This means that for any water mass for which we have information on, or we can make reasonable assumptions about, the values of a, b and $\bar{\mu}_s$ in the appropriate wavebands, we can arrive at realistic estimates of the volume of water significantly illuminated by, and thus the ecological importance of, particular bioluminescent events. These mathematical relationships are also directly relevant to any assessment of the feasibility of detection of bioluminescence signals from a distance. Of the three IOP whose values we require, it is $\bar{\mu}_s$ which presents the greatest difficulty: only a few values are available in the published literature, but current developments in optical instrumentation may remedy this. While some recent measurements suggest a greater variability in oceanic scattering phase functions than had previously been encountered, it nevertheless seems likely that the range of $\bar{\mu}_s$ values used (Figure 2) to derive the value of 1.5 for the coefficient G in equation (14) will more than encompass the values that actually occur in ocean waters. No natural surface water would ever have $\bar{\mu}_s$ as low as 0.660, the value obtained for seawater filtered for 18 hours, and at the other extreme it will be observed that the trend line in Figure 2 is heading for the origin, which of course corresponds to $\bar{\mu}_s$ = 1.0.

[20] The Monte Carlo calculations show that the attenuation of irradiance with distance from the source, of this radially propagating light flux, exhibits the same kind of dependence on scattering and absorption as the attenuation of a planar flux entering the ocean from above. Equations (12) and (14) have the same form, except that $\bar{\mu}_o$, which for a point source is always equal to 1.0, disappears in the latter. The sensitivity to scattering is, however, lower in the case of the point source. The value of G, the coefficient which determines the contribution of scattering to attenuation, is typically just over half the value for a point source as for a planar flux. For example, in the case of water with the San Diego phase function, G for vertically incident light has a value of 0.231 [*Kirk*, 1991], whereas for a point source it is equal to 0.123 (Table 1). The basis for this can intuitively be understood quite simply. Consider a vertically traveling solar photon in the ocean which undergoes scattering at angle θ. To travel through a further depth interval of 1.0 metre, it must now traverse a distance of $1/\cos\theta$ metres. A photon traveling straight out from the point source, on the other hand, which also undergoes scattering at angle θ will find that the spherical surface which defines the 1.0 metre radial distance interval from the point of scattering has, so to speak, curved round to meet it so that it traverses a distance less than $1/\cos\theta$. Thus the increase in path length, which is the main mechanism by which scattering increases attenuation with distance, is less for a point source than for a planar light flux. Another reason why scattering contributes less to attenuation in the case of the point source is that any photon which is scattered backwards relative to its initial direction will, as it continues in a straight line, and if it is not absorbed or scattered again, eventually (after traversing distance $-r\cos\theta$) find itself traveling once again away from the source and thus again becoming part of the outward radiant flux. The significant general conclusion is that the extent to which absorption rather than scattering controls attenuation is substantially greater in the case of a point source than in the case of a planar light flux.

[21] To give some substance to the sensitivity of $K_E^{ab}(av)$ to changes in the IOP, it is instructive to carry out calculations for a realistic ocean water. For Bahama Is. Station 7, *Petzold* [1972] obtained (at 530 nm) a = 0.082 m^{-1}, b = 0.117 m^{-1} and a volume scattering function from which $\overline{\mu}_s$ = 0.915 may be derived. The calculated value of $K_E^{ab}(av)$ using equation (16) is 0.0891 m^{-1}. If we double the absorption coefficient $K_E^{ab}(av)$ becomes 0.1713 m^{-1}, an increase of 92%. If we double the scattering coefficient $K_E^{ab}(av)$ becomes 0.0958 m^{-1}, an increase of 7.5%. If we increase $\overline{\mu}_s$ to 0.947 (as for S. California Station 11), then $K_E^{ab}(av)$ changes to 0.0865 m^{-1}, a decrease of 3%.

[22] Measuring the absorption coefficients of ocean waters is always difficult because the values are so low, and thus in principle require a long path length. One way of achieving this is to make measurements within the ocean of the attenuation with distance of the light emitted by a point source [*Maffione et al.*, 1993; *Bauer et al.*, 1971]. The optical path in such a case can be many metres in length, so that the desired attenuation is achieved. However, the attenuation, quite apart from that due to the inverse square law, is caused by scattering as well as absorption. The theoretical analysis presented here allows us, provided we have certain information on the scattering properties of the water, to make the appropriate correction so that the true absorption coefficient, uncompromised by scattering, can be obtained.

[23] Since scattering increases attenuation, any correction for it will always involve subtraction of some amount from the initial estimate of a. The accuracy of the correction necessarily depends in turn on the accuracy of the values of b and $\overline{\mu}_s$ used in the calculation, and here a sensitivity analysis is instructive. For the particular Southern Californian water considered above, if we vary the value of the scattering coefficient by ±10% or ±20%, the corrected value of a is still 0.032 m^{-1}. Varying b by + 30% or −30% yields a values of 0.0315 and 0.0325 m^{-1}, respectively. Sensitivity to variation in $\overline{\mu}_s$, the average cosine of scattering is, however, greater. In the six unmodified (i.e., unfiltered) water types listed in Table 1, the value of $\overline{\mu}_s$ varies from 0.867 to 0.947, corresponding to variation in $(1 - \overline{\mu}_s)$ from 0.133 to 0.053. As it happens, the S. Californian water is the one with the highest $\overline{\mu}_s$ (0.947). If our assumption about water type was incorrect to such an extent that in reality we were dealing with a water at the other end of the scale ($\overline{\mu}_s$ = 0.867), then the true corrected value of a should be 0.0297 m^{-1}, i.e., ∼0.030 m^{-1}. Thus the correction we applied (on the assumption that $\overline{\mu}_s$ = 0.947), and which gave us a = 0.032 m^{-1}, would have been only about half the true correction. Nevertheless, this value of a is still nearer the true value than the unmodified slope of the $\ln[r^2\ \vec{E}(r)]$ versus r plot (0.0337 m^{-1}). At the other extreme, if our water type was at the high end of the $\overline{\mu}_s$ scale (0.947), but we mistakenly assumed it was right down at the low end ($\overline{\mu}_s$ = 0.867), then we would overestimate the correction, and end up with a value of a which differed from the true value by about as much as did the original unmodified slope. We are of course looking here at the most extreme possible errors in our identification of water type. A practical approach if we are uncertain about $\overline{\mu}_s$, would be to assume a value which is the mean of the six values in Table 1: this in fact is 0.911. Use of this value in the correction procedure should in the vast majority of cases, by taking into account in a realistic way the contribution of scattering, yield values of absorption coefficient which are nearer the truth than the unmodified values of the slope of the $\ln[r^2\ \vec{E}(r)]$ versus r plot.

[24] It is worth noting that if we square both sides of equation (16) and rearrange

$$(1 - \overline{\mu}_s) = \frac{\left(K_E^{ab}[av]\right)^2 - a^2}{1.5ab} \tag{23}$$

then we obtain an expression by means of which if, on the basis of other measurements we have values for a, b and $K_E^{ab}(av)$ (from the $\ln[r^2\ \vec{E}(r)]$ versus r plot), then we can calculate $\overline{\mu}_s$ (I am indebted to a reviewer for drawing my attention to this). In this way we derive a value for an important but difficult to measure inherent optical property for the water mass in question.

[25] The relationships described above, by means of which we predict the light field around a bioluminescent point source, or estimate values of a or $\overline{\mu}_s$ for the water mass at particular locations in the ocean, are strictly speaking applicable only to a water column which is optically homogeneous within the depth interval of interest. Thus they will apply within the mixed layer, but will be less appropriate where, within the depth interval, there is horizontal layering of IOP due, for example, to the presence of a deep chlorophyll maximum. Errors in estimation of a due to such inhomogeneity would be minimized if the measurements were made at constant depth (and therefore within the same layer) rather than, for example, as in the *Maffione et al.* [1993] study, with varying source depth at a given detector depth. This, however, would be technically more difficult. In reality, the point source procedure examined in this study should only be applied where there is substantial optical homogeneity within the measurement depth. In the case of a bioluminescent point source located, for example, in the mixed layer just above the deep chlorophyll maximum, transmission of light both upwards and downwards would be (approximately) in accordance with equation (21), but with different values for the IOP in each case.

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